TOPIC P1: PROBABILITY - A MEASUREMENT OF UNCERTAINTY

SPOTLIGHT: HOW RISKY IS …?

The magazine *Discover* once had a special issue on "Life at Risk." In an article, Jeffrey Kluger describes the risks of making it through one day:

"Imagine my relief when I made it out of bed alive last Monday morning. It was touch and go there for a while, but I managed to scrape through. Getting up was not the only death-defying act I performed that day. There was shaving, for example; that was no walk in the park. Then there was showering, followed by leaving the house and walking to work and spending eight hours at the office. By the time I finished my day -- a day that also included eating lunch, exercising, going out to dinner, and going home -- I counted myself lucky to have survived in one piece."

Is this writer unusually fearful? No. He has read mortality studies and concludes "there is not a single thing you can do in an ordinary day -- sleeping included -- that isn't risky enough to be the last thing you ever do." In the *Book of Risks* by Laudan, we learn that

- 1 out of 2 million people will die from falling out of bed.
- 1 out of 400 will be injured falling out of bed.
- 1 out of 77 adults over 35 will have a heart attack this year.
- The average American faces a 1 in 13 risk of suffering some kind of injury in home that necessitates medical attention.
- 1 out of 7000 will experience a shaving injury requiring medical attention.
- The average American faces a 1 out of 14 risk of having property stolen this year.
- 1 out of 32 risk of being the victim of some violent crime.

- The annual odds of dying in any kind of motor vehicle accident is 1 in 5800.

Where do these reported odds come from? There are simply probabilities calculated from the counts of reported accidents. Since all of these accidents are possible, that means that there is a risk to the average American that they will happen to him or her. But fortunately, you need not worry – many of these reported risks are too small to really take seriously or change your style of living.

PREVIEW

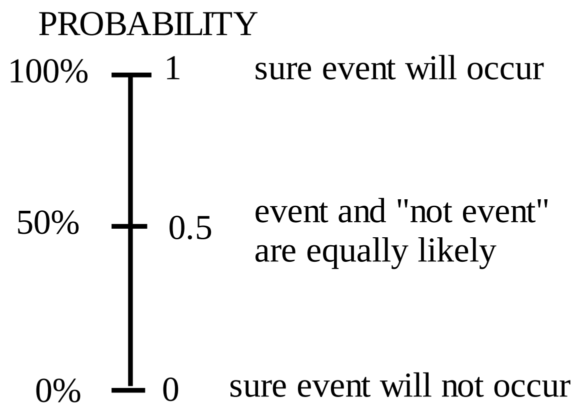
Everywhere we are surrounded by uncertainty. If you think about it, there are a number of things that you are unsure about, like

- what is the high temperature next Monday?
- how many inches of snow will our town get next January?
- what 's your final grade in this class?
- will you be living in the same state twenty years from now?
- who will win the U.S. presidential election in 2008?
- is there life on Mars?

A probability is simply a number between 0 and 1 that measures the uncertainty of a particular event.

Although many events are uncertain, we possess different degrees of belief about the truth of an uncertain event. For example, most of us are pretty certain of the statement "the sun will rise tomorrow", and pretty sure that the statement "the moon is made of green cheese" is false.

We can think of a probability scale from 0 to 1.



We would give the statement "the sun will rise tomorrow" a probability close to 1, and the statement "the moon is made of green cheese" a probability close to 0. It is harder to assign probabilities to uncertain events that have probabilities between 0 and 1. In this topic, we first get some experience in assigning probabilities. Then we will discuss three general ways of thinking about probabilities.

In this topic your learning objectives are to:

- Understand the three interpretations of probability.
- Understand what interpretations of probability are appropriate in a given situation.
- Understand how to compute approximate probabilities given data from repeated experiments.
- Understand how to obtain subjective probabilities by use of a calibration experiment.

WARM-UP ACTIVITY – SOME QUESTIONS ON PROBABILITY

For each of the following questions,

- specify the probability (as best as you can) and
- explain why you gave this particular probability value.

1. Suppose you have a bag with 4 white and 8 red balls. You choose a ball at random from the bag. What is the probability that the ball you chose is white?

2. Suppose you walk into your college bookstore blindfolded and bump into a fellow student. What is the probability the student is female?

3. ___________ [YOUR TEAM] is playing ________ [ANOTHER TEAM] soon in _________ [GIVE SPORT]. What is the chance that your team will win?

4. You drop a thumbtack 20 times on the floor and it lands with the point-side up 12 times. What is the probability that the tack will land point-side up?

5. Suppose you toss a coin 20 times and get 19 heads. What is the chance that the next toss is heads?

6. What is the chance that you will be married when you are 25 years old?

7. If you roll two dice, what is the probability that the sum of the two dice rolls is equal to 5?

8. Suppose you are going to interview a high school math teacher. What is the chance that this teacher is male?

9. What is the chance that you will complete your college education (that is, graduate) in five years or less?

10. When a meteorologist reports that there is a 50% chance of rain tomorrow, what does this mean?

11. What's the chance that two people in our class have the same birthday (month and day)?

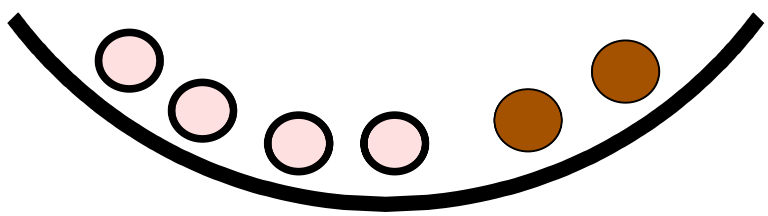
After working this activity, you should realize that probabilities are hard to measure. But there are three ways of thinking about probabilities, the classical, frequency, and subjective viewpoints, that are helpful in this measurement problem. We discuss each interpretation of probabilities in the remainder of this topic.

THE CLASSICAL VIEW OF A PROBABILITY

Suppose that we observe some phenomena (say, the rolls of two dice) where the outcome is random. Suppose we can write down the list of all possible outcomes, and we believe that each outcome in the list has the same probability. Then the probability of each outcome will be

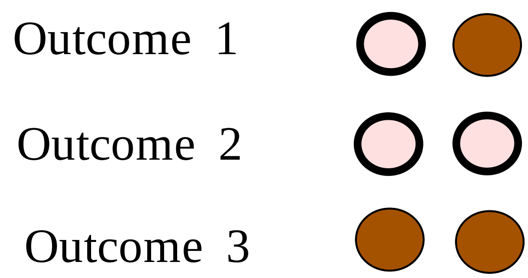
$$P(outcome) = \frac{1}{number\ of\ outcomes}.$$

Let's illustrate this classical view of probability by a simple example. Suppose you have a bowl with 4 white and 2 red balls

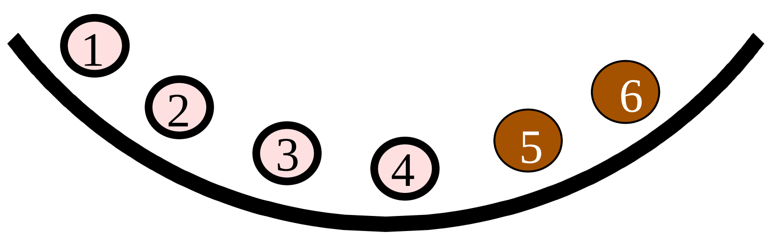


and you draw two balls from the bowl at random. We assume that the balls are drawn *without replacement* which means that you don't place a ball back into the bowl after it has been selected. What are possible outcomes? There are different ways of writing down the possible outcomes, depending if you decide to distinguish the balls of the same color.

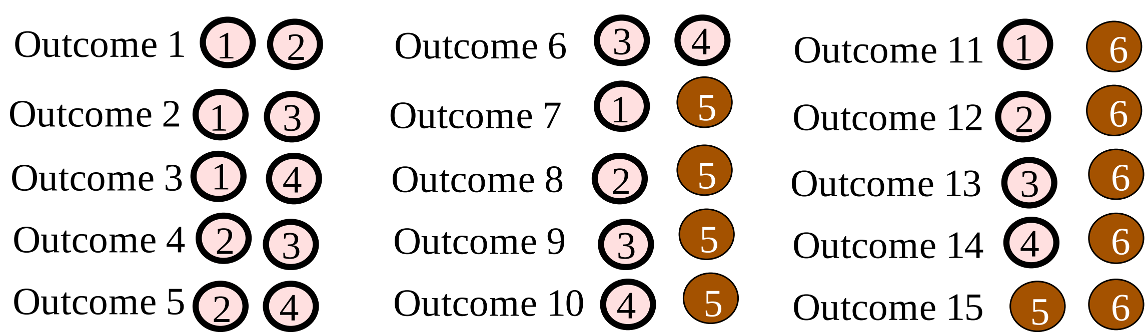
WAY 1: If we ***don't distinguish*** between balls of the same color, then there are three possible outcomes – essentially we choose 0 red, 1 red, or 2 red balls.



WAY 2: If we ***do distinguish*** between the balls of the same color, we label the balls in the bowl



and then we can write down 15 distinct outcomes of the experiment of choosing two balls.



Which is the more appropriate way of listing outcomes?

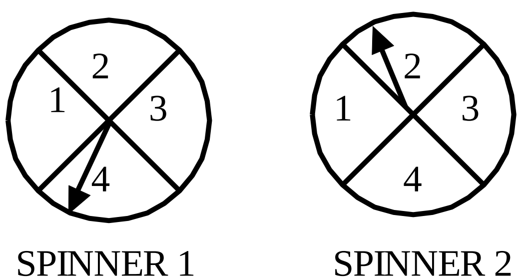
To apply the classical view of probability, we have to assume that the outcomes are all equally likely. In the first list of three outcomes, we can't assume that they are equally likely. Since there are more white than red balls in the basket, it is more likely to choose two white balls ( ) than to choose two red balls ( ). So it is incorrect to say that the probability of each one of the three possible outcomes is 1/3.

That is, the probabilities of choosing 0 red, 1 red, and 2 reds are not equal to 1/3, 1/3, and 1/3.

On the other hand, since we are choosing two balls at random from the basket, it makes sense that the 15 outcomes in the second listing (where we assumed the balls distinguishable) are equally likely. So we can apply the classical notion and assign a probability of 1/15 to each of the possible outcomes. In particular, the probability of choosing two red balls (which is one of the 15 outcomes) is equal to 1/15.

PRACTICE: THE CLASSICAL VIEW OF A PROBABILITY

Suppose you have two spinners, shown below.



(a) Suppose you spin both spinners and record the ***sum*** of the two spins. For example, if SPINNER 1 lands "2" and SPINNER 2 lands "3", the sum of the two spins is 2 + 3 = 5. Write down all of the possible sums of two spins.

(b) Assume that the possible outcomes in part (a) are equally likely. Then what would be the probability that the sum of spinners is equal to 2?

(c) Suppose instead that you record the ***Spinner 1 result and the Spinner 2 result***. (One possibility is that Spinner 1 lands 2 and Spinner 2 lands 3.) Write down all of the possible outcomes below of the two spinners. (You should have a list of 16 outcomes.)

SPINNER 1 SPINNER 2

(d) If the outcomes in (c) are assumed equally likely, what would be the probability that both spinners land 1?

(e) If you compare your answers to parts (b) and (d), note that your answers are different. That is, the probability that the SUM is equal to 2 in part (b) is not equal to the probability that both spinners land 1 in part (d). Why?

(f) In part (b), we assumed that all possible SUMS were equally likely, and in part (d), we assumed that all possible values of (SPINNER1, SPINNER2) were equally likely. Which assumption is *not* correct? Why?

THE FREQUENCY VIEW OF A PROBABILITY

The classical view of probability is helpful only when we can construct a list of outcomes of the experiment in such a way where the outcomes are equally likely.

The frequency interpretation of probability can be used in cases where outcomes are equally likely or not equally likely.

This view of probability is appropriate in the situation where we are able to ***repeat the random experiment many times under the same conditions.***

Getting out of jail in Monopoly

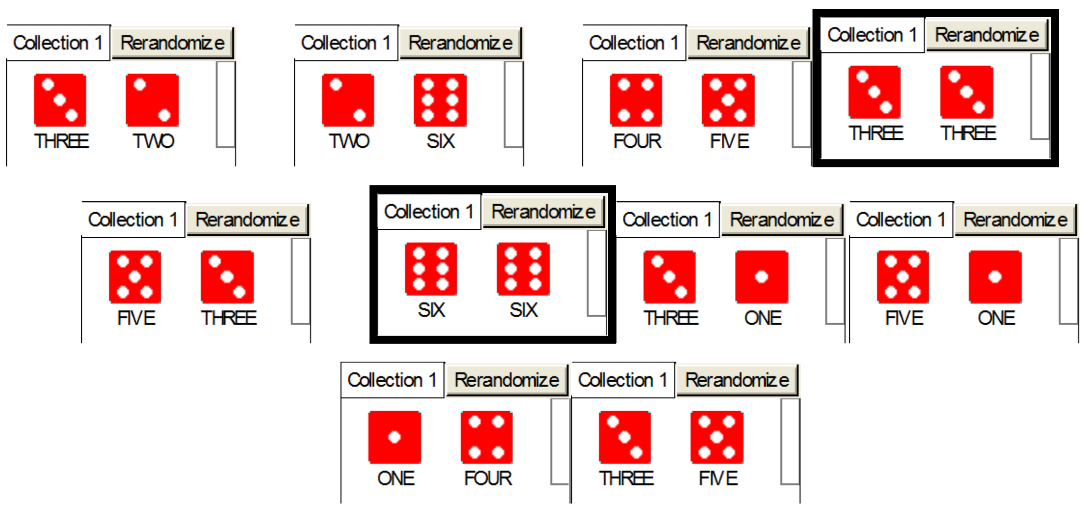
Suppose we are playing the popular game Monopoly and we land in jail. To get out of jail on the next turn, we can either pay $50 or roll "doubles" when we roll two fair dice. Doubles means that the faces on the two dice are the same. If we think that it is relatively likely to roll doubles, then we may elect to roll two dice instead of paying $50 to get out of jail.

What is the probability of rolling doubles when you roll two dice?

In this situation, we can use the frequency notion to approximate the probability of rolling doubles. We can imagine rolling two dice many times under similar conditions. Each time we roll two dice, we observe if we get doubles or not. Then the probability of doubles can be approximated by the relative frequency

Prob(doubles) = (# of doubles)/(number of experiments).

We used *Fathom* to simulate the rolling of two dice -- the results of the first 10 experiments are shown in the table below. For each experiment, we record if there is a match or no match in the two numbers that are rolled.



In these first 10 experiments, we note that we obtained a match (doubles) exactly two times. (Those are the outcomes that are boxed.) So

Prob(match) is approximately 2/10 = 0.2.

What happens if we roll the two dice more times? Then our approximate probability, the relative frequency, will get closer to the actual probability of doubles. Let us illustrate this by rolling two dice on Fathom 100 times, and then 10,000 times.

The table below tabulates the results of 100 rolls of two dice. For each roll of dice, we indicate if there was a match (YES) or not (NO).

Die 1	Die 2	Match?	Die 1	Die 2	Match?	Die 1	Die 2	Match?	Die 1	Die 2	Match?
1	6	NO	1	6	NO	6	4	NO	1	6	NO
1	3	NO	4	3	NO	5	3	NO	3	5	NO
3	3	YES	2	1	NO	6	4	NO	2	2	YES
5	4	NO	5	1	NO	6	6	YES	1	4	NO
6	5	NO	1	5	NO	6	5	NO	1	1	YES
2	2	YES	3	2	NO	2	2	YES	1	6	NO
6	2	NO	4	2	NO	4	6	NO	3	1	NO
1	2	NO	5	2	NO	1	3	NO	5	6	NO
3	2	NO	4	5	NO	3	4	NO	6	3	NO
2	6	NO	3	5	NO	4	2	NO	5	4	NO
4	6	NO	1	1	YES	2	3	NO	3	3	YES
4	4	YES	2	1	NO	1	5	NO	6	3	NO
6	4	NO	5	2	NO	2	2	YES	2	6	NO
3	5	NO	3	4	NO	4	3	NO	5	6	NO
2	1	NO	3	5	NO	5	5	YES	2	5	NO
1	1	YES	1	2	NO	1	1	YES	5	3	NO
4	3	NO	2	4	NO	6	5	NO	3	1	NO
5	1	NO	5	1	NO	2	1	NO	6	3	NO
3	4	NO	5	4	NO	3	4	NO	1	2	NO
2	1	NO	1	3	NO	2	3	NO	6	4	NO
6	5	NO	4	4	YES	6	4	NO	3	2	NO
1	1	YES	2	1	NO	1	5	NO	3	4	NO
6	4	NO	6	4	NO	2	5	NO	4	6	NO
5	1	NO	3	3	YES	6	5	NO	2	2	YES
4	3	NO	1	6	NO	2	6	NO	4	4	YES

We see from the table that we observed a match 18 times (there are 18 Yeses in the table), so

Prob(match) is approximately 18/100 = 0.18.

Let's now roll the two dice 10,000 times on computer -- this time, we observe 1662 matches, so

Prob(match) is approximately 1662/10000 = 0.1662.

Approximate and actual probabilities

Is 0.1662 the actual probability of getting doubles? No, it is still only an approximation to the actual probability.

However, as we continue to roll dice, the relative frequency

(number of doubles)/(number of experiments)

will approach the actual probability Prob(doubles).

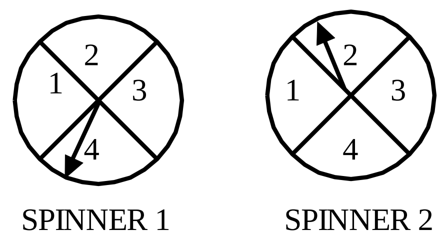
Here the actual probability of rolling doubles is

Prob(doubles) = 1/6,

which is very close to the relative frequency of doubles that we obtained by rolling the dice 10,000 times. (In this example, one can show that are 6 x 6 = 36 equally likely ways of rolling two distinguishable dice and there are exactly six ways of rolling doubles. So using the classical viewpoint, the probability of doubles is 6/36 =1/6.)

PRACTICE: THE FREQUENCY VIEW OF A PROBABILITY

Recall the spinner example considered earlier, where we spun the following two spinners:

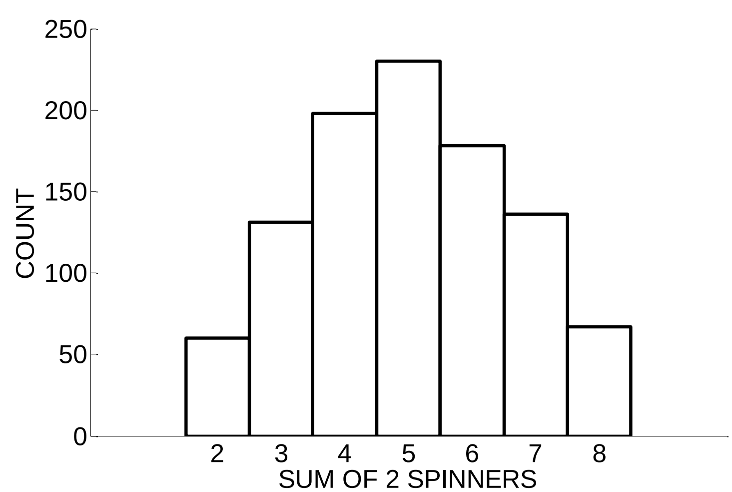


(a) Suppose we spin the two spinners 20 times with the following results. (A "3" on Spinner 1 and a "2" on Spinner 2 is represented by (3, 2).)

(4, 1)	(4, 1)	(4, 2)	(4, 2)	(4, 4)
(1, 2)	(4, 1)	(2, 1)	(3, 4)	(4, 3)
(3, 4)	(2, 1)	(3, 1)	(1, 2)	(2, 1)
(2, 1)	(1, 3)	(4, 3)	(2, 2)	(4, 3)

Using these data, compute the (approximate) probability the SUM of the two spinners is equal to 5.

(b) The two spinners were spun 1000 times – each time, the SUM of the two spins was recorded. The histogram below displays the results of the 1000 experiments.



Using the histogram, compute the approximate probability the SUM of the two spinners is equal to 5.

(c) The table below shows all of the possible outcomes of rolling the two spinners and the value of the sum S for each outcome. Assuming that the outcomes are equally likely, find the actual probability S is equal to 5. Compare your answer with the approximate probability from part (b).

Outcome	S	Outcome	S	Outcome	S	Outcome	S
(1, 1)	2	(2, 1)	3	(3, 1)	4	(4, 1)	5
(1, 2)	3	(2, 2)	4	(3, 2)	5	(4, 2)	6
(1, 3)	4	(2, 3)	5	(3, 3)	6	(4, 3)	7
(1, 4)	5	(2, 4)	6	(3, 4)	7	(4, 4)	8

ACTIVITY: TOSSING AND SPINNING A POKER CHIP

To apply the frequency notion of probability, it is important that we are able to perform our random experiment repeatedly under similar conditions. It might seem that this is obvious, but actually it is hard to repeat an experiment exactly the same way. We illustrate this through the simple experiment of flipping a poker chip.

MATERIALS NEEDED: A set of standard plastic poker chips and a container of silly putty or gum. (If poker chips are not available, you can do these experiments with all quarters or all nickels.)

1. Look at your poker chip and decide which side is “heads”. Consider the experiment of flipping a poker chip 20 times and counting the number of heads. Before you do this, the instructor and class should decide exactly what it means to “flip a chip.” Here are some guidelines for flipping (the class may wish to adjust these rules):

- you flip the chip in the air
- the chip should flip at least five times in the air before coming down
- the chip has to land on the desk
- any experiments with insufficient flips or where the chip falls on the floor are ignored

First try out your flipping method until you have a style that you can repeat many times. Then flip the chip 20 times and record the number of heads you observe.

2. Next, consider the experiment of spinning the chip 20 times and recording the number of heads. To spin the chip, you hold the chip level with one finger and flick the chip with the other finger so that the chip spins on the table with at least five spins. Practice spinning until you are comfortable with the method. Then spin the chip 20 times, recording the number of heads.

3. Suppose you add a small amount of putty to the heads side of the chip. Do you think this change will modify the probability of heads when flipping? What if you spin – do you think the probability of heads will change?

4. Put the putty on the heads side and flip the chip 20 times – record the number of heads.

5. Now spin the modified chip 20 times – record the number of heads.

6. Combine the class data for the four experiments – flip regular chip, spin regular chip, flip modified chip, and spin modified chip. For each dataset, construct a graph of the number of heads, and find a typical number of heads. (You will have four graphs and summary values corresponding to the four experiments.)

7. Summarize your results. What is the probability of heads when the chip is flipped? What is the probability when the chip is spun? Does the putty affect the probability of heads for the flipped chip? Does the putty change the probability of heads on a spin?

THE SUBJECTIVE VIEW OF A PROBABILITY

We have described two ways of thinking about probabilities:

The classical view. This is a useful way of thinking about probabilities when one can list all possible outcomes in such a way that each outcome is equally likely.

The frequency view. In the situation when you can repeat a random experiment many times under similar conditions, you can approximate a probability of an event by the relative frequency that the event occurs.

What if you can't apply these two interpretations of probability? That is, what if the outcomes of the experiment are not equally likely, and it is not feasible or possible to repeat the experiment many times under similar conditions?

In this case, we can rely on a third view of probabilities, ***the subjective view***. This interpretation is arguably the most general way of thinking about a probability, since it can be used in a wide variety of situations.

Suppose you are interested in the probability of the event

"Your team will win the conference title in basketball next season."

You can't use the classical or frequency views to compute this probability. Why?

Suppose there are eight teams in your conference. Each team is a possible winner of the conference, but these teams are not equally likely to win -- some teams are stronger than the rest. So the classical approach won't help in obtaining this probability.

The event of your team winning the conference next year is essentially a one-time event. Certainly, your team will have the opportunity to win this conference in future years, but the players on your team and their opponents will change and it won't be the same basketball competition. So you can't repeat this experiment under similar conditions, and so the frequency view is not helpful in this case.

What is a subjective probability in this case? The probability

Prob(Your team will win the conference in basketball next season)

represents your belief in the likelihood that your team will win the basketball conference next season. If you believe that your school will have a great team next year and will win most of their conference games, you would give this probability a value close to 1. On the other hand, if you think that your school will have a relatively weak team, your probability of this event would be a small number close to 0. Essentially, this probability is a numerical statement about your confidence in the truth of this event.

There are two important aspects of a subjective probability.

1. A subjective probability is ***personal***. Your belief about your team winning the basketball conference is likely different from my belief about your team winning the conference since we have different information. Perhaps you are not interested in basketball and know little about the teams and I am very knowledgeable about college basketball. That means that our beliefs about the truth of this event will be different and so our probabilities would also be different.

2. A subjective probability ***depends on your current information*** or knowledge about the event in question. Maybe you originally think that this probability is .7 since your school had a good team last year. But when you learn that many of the star players from last season have graduated, this changes your knowledge about the team, and you may now assign this probability a smaller number.

Measuring Probabilities Subjectively

Although we are used to expressing our opinions about uncertain events, using words like

likely, probably, rare, sure, maybe,

we are *not* used to assigning probabilities to quantify our beliefs about these events. To make any kind of measurement, we use a tool like a scale or ruler. Likewise, we need tools to help us assign probabilities subjectively. In the next activity, we illustrate a special tool, called a *calibration experiment*, that will help to determine our subjective probabilities.

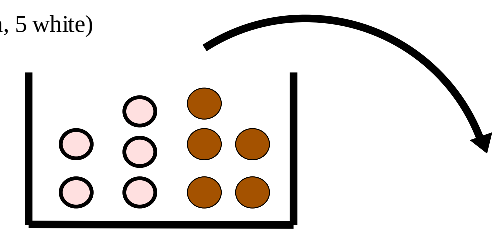
A CALIBRATION EXPERIMENT

Consider the event

W = "a woman will be President of the United States in the next 20 years".

We are interested in your subjective probability of W. This probability is hard to specify precisely since we haven't had much practice doing it. We describe a simple procedure that will help in measuring this probability.

First consider the following calibration experiment – this is an experiment where the probabilities of outcomes are clear. We have a collection of balls, 5 red and 5 white in a box and we select one ball at random.



Let B denote the event that we observe a red ball. Since each of the ten balls is equally likely to be selected, I think we would agree that Prob(B) = 5/10 = .5.

Now consider the following two bets:

- BET 1 – If W occurs (a women is president in the next 20 years), you win $100. Otherwise, you win nothing.
- BET 2 – If B occurs (you observe a red ball in the above experiment), you win $100. Otherwise, you win nothing.

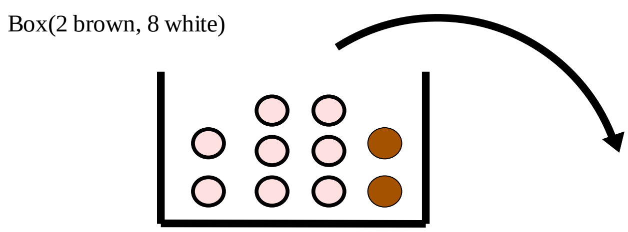
Based on the bet that you prefer, we can determine an interval that contains your Prob(W):

- If you prefer BET 1, then your Prob(W) must be larger than Prob(B) = .5 – that is, your Prob(W) must fall between .5 and 1.

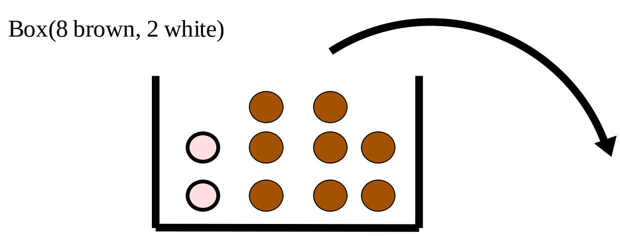
- If you prefer BET 2, then your Prob(W) must be smaller than Prob(B) = .5 – that is, your probability of W must fall between 0 and .5.

What you do next depends on your answer to part (b).

- <u>If your Prob(W) falls in the interval (0, .5),</u> then consider the "balls in box" experiment with 2 red and 8 white balls and you are interested in the probability of choosing a red ball.



- <u>If instead your Prob(W) falls in the interval (.5, 1),</u> then consider the "balls in box" experiment with 8 red and 2 white balls and you are interested in the probability of choosing a red ball.



Let's suppose that you believe Prob(W) falls in the interval (.5, 1). Then you would make a judgment between the two bets

- BET 1 – If W occurs (a women is president in the next 20 years), you win $100. Otherwise, you win nothing.
- BET 2 – If B occurs (you observe a red ball with a box with 8 red and 2 white balls), you win $100. Otherwise, you win nothing.

I decide to prefer BET 2, which means that my probability Prob(W) is smaller than 0.8. Based on the information on the two comparisons, you now believe that Prob(W) falls between .5 and .8.

In practice, you will continue to compare BET 1 and BET 2, where the box has a different number of red and white balls. By a number of comparisons, you will get an accurate measurement at your probability of W.

PRACTICE: A CALIBRATION EXPERIMENT

1. Consider the following “balls in box” experiments. What is the probability of drawing a red if the box contains

(a) 5 red and 5 white?
(b) 2 red and 8 white?
(c) 7 red and 3 white?
(d) 0 red and 10 white?

2. Consider the statement

A: “I will get married in the next five years"

We want to determine your personal probability that A is true, call this PROB(A).

(If you are already married, choose a different statement where the truth of the statement is uncertain.)

Consider the following two bets:

BET 1:

- If you are married in the next five years, then you win $20.
- If you are not married in the next five years, you win nothing.

BET 2:

- If you draw a red in a balls-in-box experiment with 5 red and 5 white balls, then you win $20.
- If you draw a white, you win nothing.

(a) Which bet (BET 1 or BET 2) do you prefer?

(b) Based on your answer to (a), do you think PROB(A) > .5 or PROB(A) < .5?

3. Let's continue to make this comparison for more balls-in-box experiments. Each row of the table below gives two choices. The left choice is BET 1: you win $20 if you are married in five years and win nothing if this event does not happen. The choice on the right is BET 2: you win $20 if you draw a red from a box with a certain number of reds and whites; otherwise you win nothing. For each pair of bets, circle the choice which you prefer

BET 1	BET 2
$20 if you are married in five years Nothing if you are not married in five years	$20 if draw red in box with 0 red, 10 white Nothing if draw white
$20 if you are married in five years Nothing if you are not married in five years	$20 if draw red in box with 2 red, 8 white Nothing if draw white
$20 if you are married in five years Nothing if you are not married in five years	$20 if draw red in box with 4 red, 6 white Nothing if draw white

\$20 if you are married in five years Nothing if you are not married in five years	\$20 if draw red in box with 6 red, 4 white Nothing if draw white
\$20 if you are married in five years Nothing if you are not married in five years	\$20 if draw red in box with 8 red, 2 white Nothing if draw white
\$20 if you are married in five years Nothing if you are not married in five years	\$20 if draw red in box with 10 red, 0 white Nothing if draw white

Based on your choices you made above, you should have a better idea about your probability of being married in five years. Mark on the below number line your probability. (This value should be consistent with the choices that you made.)

```
+----+----+----+----+----+----+----+----+----+----+-
0.0  0.1  0.2  0.3  0.4  0.5  0.6  0.7  0.8  0.9  1.0
MY PROBABILITY OF BEING MARRIED IN FIVE YEARS
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4. Repeat the process in part 3 with a different statement.

PRACTICE: VIEWPOINTS OF PROBABILITY

Consider again the probability questions considered in the opening activity. For each question, ***give the viewpoint*** (classical, frequency, or subjective) that is helpful in obtaining the probability and ***explain how you use*** the viewpoint in the calculation.

1. Suppose you have a bag with 4 white and 8 red balls. You choose a ball at random from the bag. What is the probability that the ball you chose is white?
2. Suppose you walk into your college bookstore blindfolded and bump into a fellow student. What is the probability the student is female?
3. ____________ [YOUR TEAM] is playing _________ [ANOTHER TEAM] soon in _________ [GIVE SPORT]. What is the chance that your team will win?

4. You drop a thumbtack 20 times on the floor and it lands with the point-side up 12 times. What is the probability that the tack will land point-side up?

5. Suppose you toss a coin 20 times and get 19 heads. What is the chance that the next toss is heads?

6. What is the chance that you will be married when you are 25 years old?

7. If you roll two dice, what is the probability that the sum of the two dice rolls is equal to 5?

8. Suppose you are going to interview a high school math teacher. What is the chance that this teacher is male?

9. What is the chance that you will complete your college education (that is, graduate) in five years or less?

WRAP-UP

In this topic, we were introduced to three different ways of thinking about probabilities. The classical viewpoint of probability allows one to compute probabilities based on the structure of the problem. To apply this viewpoint, it is convenient to write down the collection of possible outcomes in such a way that each outcome is equally likely to occur. The frequency viewpoint is useful in the situation where one can repeat the random experiment many times under identical conditions, and the probability of an event is approximately equal to the fraction of experiments where the event occurs. The subjective viewpoint is helpful in the situation where one expresses his or her opinion about the likelihood of a one-time event. A calibration experiment provides a means for measuring subjective probabilities by comparing the situation with a different experiment with known probabilities. These three viewpoints allow us to express uncertainty in a wide variety of situations.

EXERCISES

1. **Probability Viewpoints**

In the following problems, indicate if the given probability is found using the classical viewpoint, the frequency viewpoint, or the subjective viewpoint.

a. Joe is doing well in school this semester – he is 90 percent sure that he will receive an A in all of his classes.

b. Two hundred raffle tickets are sold and one ticket is a winner. I purchased one ticket and the probability that my ticket is the winner is 1/200.

c. Suppose that 30% of all college women are playing an intercollegiate sport. If I contact one college woman at random, the chance that she plays a sport is .3.

d. Two Polish statisticians in 2002 were questioning if the new Belgium Euro coin was indeed fair. They had their students flip the Belgium Euro 250 times, and 140 came up heads.

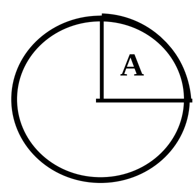
e. Many people are afraid of flying. But over the decade 1987-96, the death risk per flight on a US domestic jet has been 1 in 7 million.

f. In a roulette wheel, there are 38 slots numbered 0, 00, 1, …, 36. There are 18 ways of spinning an odd number, so the probability of spinning an odd is 18/38.

2. **Probability Viewpoints**

In the following problems, indicate if the given probability is found using the classical viewpoint, the frequency viewpoint, or the subjective viewpoint.

a. The probability that the spinner lands in the region A is ¼.



b. The meteorologist states that the probability of rain tomorrow is .5. You think it is more likely to rain and you think the chance of rain is ¾.

c. A football fan is 100% certain that his high school football team will win their game on Friday.

d. Jennifer attends a party, where a prize is given to the person holding a raffle ticket with a specific number. If there are eight people at the party, the chance that Jennifer wins the prize is 1/8.

e. What is the chance that you will pass this English class? You learn that the professor passes 70% of the students and you think you are typical in ability among those attending the class.

f. If you toss a plastic cup in the air, what is the probability that it lands with the open side up? You toss the cup 50 times and it lands open side up 32 times, so you approximate the probability by 32/50

3. **Equally Likely Outcomes**

For the following experiments, a list of possible outcomes is given. Decide if one can assume that the outcomes are equally likely. If the equally likely assumption is not appropriate, explain which outcomes are more likely than others.

a. A bowl contains six marbles of which two are red, three are white, and one is black. One marble is selected at random from the bowl and the color is observed.

Outcomes: {red, white, black}

b. You observe the gender of a baby born today at your local hospital.

Outcomes: {male, female}

c. Your school's football team is playing the top rated school in the country.

Outcomes: {your team wins, your team loses}

d. A bag contains 50 slips of paper, 10 that are labeled "1", 10 labeled "2", 10 labeled "3", 10 labeled "4", and 10 labeled "5". You choose a slip at random from the bag and notice the number on the slip.

Outcomes: {1, 2, 3, 4, 5}

4. **Equally Likely Outcomes**

For the following experiments, a list of possible outcomes is given. Decide if one can assume that the outcomes are equally likely. If the equally likely assumption is not appropriate, explain which outcomes are more likely than others.

a. You wait at a bus stop for a bus. From experience, you know that you wait, on average, 8 minutes for this bus to arrive.

Outcomes: {wait less than 10 minutes, wait more than 10 minutes}

b. You roll two dice and observe the sum of the numbers.

Outcomes: {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

c. You get a grade for a English course in college.

Outcomes: {A, B, C, D, F}

d. You interview a person at random at your college and ask for his or her age.

Outcomes: {17 to 20 years, 21 to 25 years, over 25 years}

5. **Flipping a Coin**

Suppose you flip a fair coin until you observe heads. You repeat this experiment many times, keeping track of the number of flips it takes to observe heads. Here are the numbers of flips for 30 experiments.

1	3	1	2	1	1	2	6	1	2
1	1	1	1	3	2	1	1	2	1
5	2	1	7	3	3	3	1	2	3

a. Approximate the probability that it takes you exactly two flips to observe heads.

b. Approximate the probability that it takes more than two flips to observe heads.

c. What is the most likely number of flips?

6. **Driving to Work**

You drive to work 20 days, keeping track of the commuting time (in minutes) for each trip. Here are the twenty measurements.

25.4, 27.8, 26.8, 24.1, 24.5, 23.0, 27.5, 24.3, 28.4, 29.0

29.4, 24.9, 26.3, 23.5, 28.3, 27.8, 29.4, 25.7, 24.3, 24.2

a. Approximate the probability that it takes you under 25 minutes to drive to work.

b. Approximate the probability it takes between 25 and 28 minutes to drive to work.

c. Suppose one day it takes you 23 minutes to get to work. Would you consider this unusual? Why?

7. **A Man Sent to the Moon**

Consider your subjective probability P(M) where M is the event that the United States will send a man to the moon in the next twenty years.

a. Let B denote the event that you select a red ball from a box of five red and five white balls. Consider the two bets

- BET 1 – If M occurs (United States will send a man to the moon in the next 20 years), you win $100. Otherwise, you win nothing.
- BET 2 – If B occurs (you observe a red ball in the above experiment), you win $100. Otherwise, you win nothing.

Circle the bet that you prefer.

b. Let B represent choosing red from a box of 7 red and 3 white balls. Again compare BET 1 with BET 2 – which bet do you prefer?

c. Let B represent choosing red from a box of 3 red and 7 white balls. Again compare BET 1 with BET 2 – which bet do you prefer?

d. Based on your answers to (a), (b), (c), circle the interval of values that contain your subjective probability P(M).

0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1

8. **What State Will You Be Living in the Future?**

Consider your subjective probability P(S) where S is the event that at age 30 you will be living in the same state as you currently live. Let B denote the event that you select a red ball from a box of five red and five white balls. Consider the two bets

- BET 1 – If M occurs (you live in the same state at age 30), you win $100. Otherwise, you win nothing.
- BET 2 – If B occurs (you observe a red ball in the above experiment), you win $100. Otherwise, you win nothing.

Circle the bet that you prefer.

b. Let B represent choosing red from a box of 7 red and 3 white balls. Again compare BET 1 with BET 2 – which bet do you prefer?

c. Let B represent choosing red from a box of 3 red and 7 white balls. Again compare BET 1 with BET 2 – which bet do you prefer?

d. Based on your answers to (a), (b), (c), circle the interval of values that contain your subjective probability P(M).

0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1

9. **Frequency of Vowels in *Huckleberry Finn***

Suppose you choose a page at random from the book *Huckleberry Finn* by Mark Twain and find the first vowel on the page.

a. If you believe it is equally likely to find any one of the five possible vowels, fill in the probabilities of the vowels below.

Vowel	a	e	i	o	u
Probability					

b. Based on your knowledge about the relative use of the different vowels, assign probabilities to the vowels.

Vowel	a	e	i	o	u
Probability					

c. Do you think it is appropriate to apply the classical viewpoint to probability in this example? (Compare your answers to parts a and b.)

d. On each of the first fifty pages of *Huckleberry Finn*, your author found the first five vowels. Here is a table of frequencies of the five vowels:

Vowel	a	E	i	o	u
Frequency	61	63	34	70	22
Probability					

Use this data to find approximate probabilities for the vowels.

10. **Purchasing Boxes of Cereal**

Suppose a cereal box contains one of four different posters denoted A, B, C, and D. You purchase four boxes of cereal and you count the number of posters (among A, B, C, D) that you *do not* have. The possible number of "missing posters" is 0, 1, 2, and 3.

a. Assign probabilities if you believe the outcomes are equally likely.

Number of missing posters	0	1	2	3
Probability				

b. Assign probabilities if you believe that the outcomes 0 and 1 are most likely to happen.

Number of missing posters	0	1	2	3
Probability				

c. Suppose you purchase many groups of four cereals, and for each purchase, you record the number of missing posters. The number of missing posters for 20 purchases is displayed below. For example, in the first purchase, you had 1 missing poster, in the second purchase, you also had 1 missing poster, and so on.

1, 1, 1, 2, 1, 1, 0, 0, 2, 1, 2, 1, 3, 1, 2, 1, 0, 1, 1, 1

Using these data, assign probabilities.

Number of missing posters	0	1	2	3
Probability				

d. Based on your work in part c, is it reasonable to assume that the four outcomes are equally likely? Why?