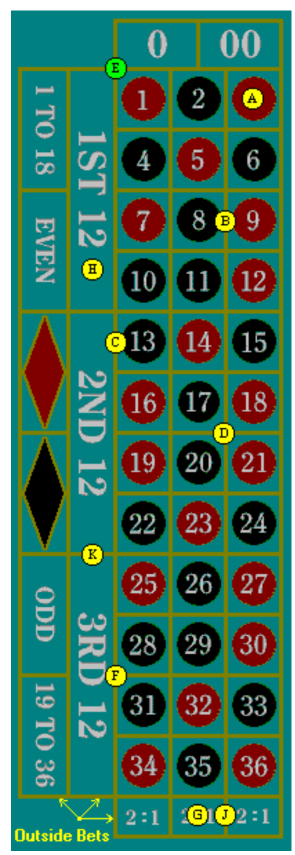
TOPIC P2: SAMPLE SPACE AND ASSIGNING PROBABILITIES

SPOTLIGHT: THE CASINO GAME OF ROULETTE

Roulette is one of the most popular casino games. The name roulette is derived from the French word meaning small wheel. Although the origin of the game is not clear, it became very popular during the 18th century when Prince Charles introduced gambling to Monaco to alleviate the country's current financial problems. The game was brought to America in the early part of the 19th century and is currently featured in all casinos. In addition, roulette is a popular game among people who like to game online.

The American version of the game that we discuss in this book varies slightly from the European version. The American roulette wheel contains 38 pockets, numbers 1 through 36 plus zero plus double zero. The wheel is spun and a small metal ball comes to rest in one of the 38 pockets.

A typical roulette table is pictured below. Players will place chips on particular locations on the table, predicting where the ball will land when after the wheel is spun and the ball comes to a stop. The dealer places a mark on the winning number. The players who have bet on the winning number are rewarded while the players who bet on losing numbers lose their chips to the casino.



In a later topic, we will introduce different types of bets that a player can make in roulette and learn about the average winnings per pay if you place a particular bet. It is important to remark that American roulette is a game that favors the casino, and a player will on average lose money by placing any type of bet. It may be a nice form of entertainment, but it certainly is not a way to earn money.

PREVIEW

A ***random experiment*** is the name for some process where the outcome is random or unknown. Examples of some random experiments are

- tossing four coins and observing the number of heads that land up
- watching cars that pass a given intersection and counting the number of cars until you see a white one
- recording the time (in minutes) it takes you to get to work
- observing the amount of money you will win on a lottery ticket bought today

Before we can talk about probabilities, we first need to get a good handle on possible outcomes of the random experiment. A list of all possible outcomes is called the ***sample space*** and denoted by S. In this topic, we describe different ways of writing down sample spaces. Probabilities are numbers assigned to outcomes of the sample space that satisfy basic rules. We will get some experience in specifying probabilities and using the rules to derive other properties about probabilities.

In this topic your learning objectives are to:

- Understand how to write down sample spaces for a variety of random experiments.
- Understand how to specify reasonable sets of probabilities satisfying certain assumptions.
- Understand basic properties of probabilities.

NCTM Standards

✓ In Grades 6-8, all students should understand and use appropriate terminology to describe complementary and mutually exclusive events.

✓In Grades 6-8, all students should compute probabilities for simple compound events, using such methods as organized lists, and tree diagrams.

✓In Grades 9-12, all students should understand the concepts of sample space and probability distribution and construct sample spaces in simple cases.

WARMUP ACTIVITY: WRITING DOWN SOME SAMPLE SPACES

For each of the following situations, write down a list of all possible outcomes.

1. TOSSING FOUR COINS. Suppose you toss four coins and you record the number of coins that show heads. What are the possible outcomes?

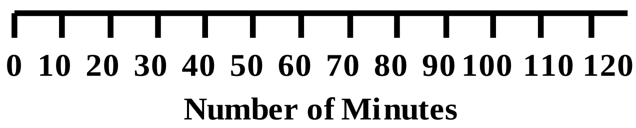
2. WATCHING CARS. Suppose you are standing at a particular intersection and you record the number of cars you observe before you see a white one. What are different outcomes for the number of cars you observe?

Note: This second example illustrates an infinite sample space, since the number of possible outcomes is infinite.

3. HOW LONG IS YOUR COMMUTE? Suppose you are a commuter and your home is located 25 miles from campus. Generally it takes you 40 minutes to get to school, but there is some variation in this commuting time. On a quiet day with little traffic, it is possible that it will only take 35 minutes to get to work. On other days, there is construction on the road and you can get stuck in traffic, and it will take close to an hour to work.

(a) Is it possible to write down all possible commuting times? Why or why not?

(b) When the outcome is continuous-valued, one convenient way of representing the sample space is by use of a number line. On the number line below, shade the region of possible commuting times.



4. WINNING IN A LOTTERY. What happens when you buy a lottery ticket? Well, your ticket could be a loser (and you win nothing), or you may win different dollar amounts depending on how closely your ticket number agrees with the winning number. Suppose that the possible winning dollar amounts are $10, $100, or $50,000 (the last prize corresponds to the big jackpot). You record the amount of money you win from one ticket. Write down the sample space.

DIFFERENT REPRESENTATIONS OF A SAMPLE SPACE

A sample space lists all possible outcomes of a random experiment. There are different ways to write down the sample space, depending on how we think about outcomes.

Let's illustrate the variety of sample spaces by the simple experiment

"roll two fair dice"

Each die is the usual six-sided object that we are familiar with, with the numbers 1, 2, 3, 4, 5, 6 on each side. When we say "fair dice", we are imagining that each die is constructed such that the six possible numbers are equally likely to come up when rolled.

What can happen when you roll two dice? The collection of all outcomes that are possible is the sample space. But there are different ways of representing the sample space depending on what "outcome" we are considering.

First, suppose we are interested in the *sum of the numbers* on the two dice. (This would be of interest to a gambler playing the casino game craps.) What are the possible sums? After some thought, it should be clear that the smallest possible sum is 2 (if you roll two ones) and the largest possible sum is 12 (with two sixes). Also every whole number between 2 and 12 is a possible sum. So the sample space, denoted by S, would be

$$S = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}.$$

Suppose instead that you wish to record *the rolls on each of the two dice*. One possible outcome would be

(4 on one die, 3 on the other die)

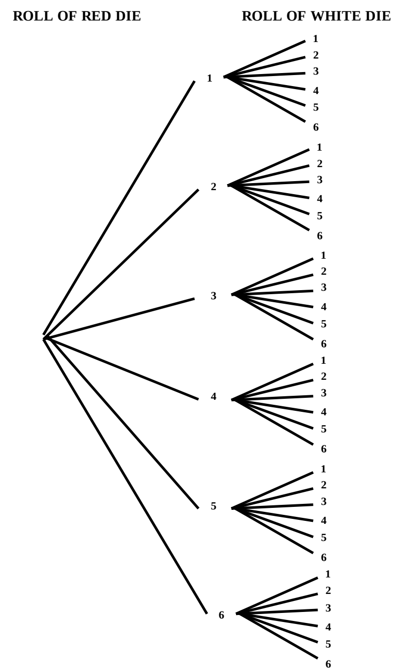
or more simply (4, 3). What are the possible outcomes? Here are the twenty-one possibilities:

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 3), (3, 4), (3, 5), (3, 6)
(4, 4), (4, 5), (4, 6)
(5, 5), (5, 6)
(6, 6)

Notice that we aren't distinguishing between the two dice in this list. For example, we just wrote down (2, 3) once, although there are two ways for this to happen - either the first die is 2 and the second die is 3 or the other way around.

Last, suppose you can distinguish the two dice -- perhaps one die is red and one die is white -- and you are considering all of the possible rolls of both dice. We illustrate three ways of showing the sample space in this case.

One way of representing possible rolls of two distinct dice is by a *tree diagram* shown on the next page. On the left side of the diagram, we represent the six possible rolls of the red die by six branches of a tree. Then, on the right side, we represent the six possible rolls of the white die by six smaller branches coming out of each roll of the red die. A single branch on the left and a single branch on the right represent one possible outcome of this experiment.



An alternative way of representing the possible outcomes of rolling two distinct dice uses a rectangular grid or table. The six possible rolls of the white die are the rows of the table, the six possible rolls of the red die correspond to the columns of the table, and the possible outcomes are represented by the x's in the table.

		Roll on red die					
		One	Two	Three	four	five	six
Roll on white die	One	x	X	X	x	x	x
	Two	x	X	X	x	x	x
	Three	x	X	X	x	x	x
	Four	x	X	X	x	x	x
	Five	x	X	X	x	x	x
	Six	x	X	X	x	x	x

There are still other ways to represent the outcomes of this experiment of rolling two distinct dice. Suppose we write down an outcome by the ordered pair

(roll on white die, roll on red die).

Then the possible outcomes are listed below.

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

Since these are ordered pairs, the *order* of the numbers does matter. The outcome (5, 1) (5 on the red, 1 on the white) is different from the outcome (1, 5) (1 on the red die and 5 on the white die).

We have illustrated three representations of the sample space of possible rolls of two dice. These representations differ by how we record the outcome of rolling two dice. We can either (1) record the sum of the two dice, (2) record the individual rolls, not distinguishing the two dice, or (3) record the individual rolls, distinguishing the two dice. Which one is the best sample space to use? Actually, all of the sample spaces shown above are correct. Each sample space represents all possible outcomes of the experiment of rolling two dice and you can't say that one sample space is better than another sample space. But we will see that in particular situations some sample spaces are more convenient than other sample spaces when we wish to assign probabilities. (In this case the sample space with distinguishable dice is desirable from the viewpoint of computing probabilities since the outcomes are equally likely.)

When we write down sample spaces, we use whatever method we like. We can use a tree diagram or a rectangular grid, or we might like to list the outcomes. The important thing is that we have shown all of the possible outcomes in S.

PRACTICE: DIFFERENT WAYS OF REPRESENTING SAMPLE SPACES

Suppose you take a survey and you ask the question: "Do you think the world is safer today than it was ten years ago?" and each person will either respond "yes" or "no". You ask this survey question to three people.

1. If you are only interested in the number of people who say yes, write down the sample space.

2. Suppose you record the answer of person 1, person 2, and person 3 with a Y for yes and an N for no. To illustrate, a "yes" by person 1, a "no" by person 2, and a "yes" by person 3 is recorded by YNY. Write down the sample space by listing all of the possible outcomes.

3. For the same situation as part 2, use a tree diagram to represent the sample space.

4. Is it reasonable to assume that the outcomes in the sample space in part 2 are equally likely? Why or why not?

ASSIGNING PROBABILITIES

When we have a random experiment, the first step is to list all of the possible outcomes in the sample space. The next step is to assign numbers, called probabilities, to the different outcomes that reflect the likelihoods that these outcomes can occur.

To illustrate different assignments of probabilities, suppose my daughter goes to an ice cream parlor and plans to order a single-dip ice cream cone. This particular parlor has four different ice cream flavors. Which flavor will my daughter order?

First, we write down the sample space -- the possible flavors that my daughter can order. Underneath we will place probabilities to these four possible outcomes that reflect my beliefs about her likes and dislikes.

	Flavor			
	Vanilla	Chocolate	Butter Pecan	Maple Walnut
Probability				

Can our probabilities be any numbers? Not exactly. Here are some basic facts (or laws) about probabilities:

- Any probability we assign must fall between 0 and 1
- The sum of the probabilities across all outcomes must be equal to 1.
- We can give an outcome a probability of 0 if we are sure that that outcome will never occur.
- Likewise, if we assign a probability of 1 to an event, then that event must occur all the time.

With these facts in mind, we consider some possible probability assignments for the flavor of ice cream that my daughter will order.

SCENARIO 1: Suppose that my daughter likes to be surprised. She has brought a hat in which she has placed many slips of paper -- 10 slips are labeled "vanilla", 10 slips are labeled "chocolate", and 10 slips are "butter pecan", and 10 are "maple walnut". She makes her ice cream choice by choosing a slip at random. In this case, each flavor would have a probability of 10/40 = ¼ .

	Flavor			
	Vanilla	Chocolate	Butter Pecan	Maple Walnut
Probability	¼	¼	¼	¼

SCENARIO 2: Let's consider a different set of probabilities based on different assumptions about my daughter's taste preferences. I know that she really doesn't like "plain" flavors like vanilla or chocolate, and she really likes ice creams with nut flavors. In this case, I would assign “Vanilla” and “Chocolate” each a probability of 0, and assign the two other flavors probabilities that sum to one. Here is one possible assignment.

	Flavor

	Vanilla	Chocolate	Butter Pecan	Maple Walnut
Probability	0	0	.7	.3

Another possible assignment of probabilities that is consistent with these assumptions is

	Flavor			
	Vanilla	Chocolate	Butter Pecan	Maple Walnut
Probability	0	0	.2	.8

SCENARIO 3: Let's consider an alternative probability assignment from a different person's viewpoint. The worker at the ice cream shop has no idea what flavor my daughter will order. But she's been working at the shop all day and she has kept a record of how many cones of each type have been ordered – of 50 cones ordered, 10 are vanilla, 14 are chocolate, 20 are butter pecan, and 6 are maple walnut. If she believes that my daughter has similar tastes to the previous customers, then it would be reasonable to apply the frequency viewpoint to assign the probabilities.

	Flavor			
	Vanilla	Chocolate	Butter Pecan	Maple Walnut
Probability	10/50	14/50	20/50	6/50

Each of the above probability assignments used a different viewpoint of probability as described in Topic P1. The first assignment used the classical viewpoint where each of the forty slips of paper had the same probability of being selected. The second assignment was an illustration of the subjective view where my assignment was based on my opinion about the favorite flavors of my daughter. The last assignment was based on the frequency viewpoint where the probabilities were estimated from the observed flavor preferences of 50 previous customers.

PRACTICE: ASSIGNING PROBABILITIES

Suppose you are interested in assigning probabilities to each of the final possible grades (A, B, C, D, and F) in one class that you are currently taking. Assign probabilities to all five grades based on the given information. (It is possible that there are multiple probability assignments that are possible. In this case, you need to give only one of the possible assignments.)

1. Assign probabilities to all 5 grades if the only possible grades are A or B.

Grade	A	B	C	D	F
Probability					

2. Assign probabilities to all 5 grades if each possible grade is equally likely.

Grade	A	B	C	D	F
Probability					

3. Suppose 200 students previously took this class – 30 got A's, 80 got B's, 50 got C's, 30 got D's, and 10 failed. If you believe you are similar in ability to these 200 students, assign probabilities.

Grade	A	B	C	D	F
Probability					

4. Assign probabilities if the only possible grades are B and C, and B is twice as likely as C.

Grade	A	B	C	D	F
Probability					

5. (Continuation of part 3.) Assign probabilities if you believe you are better than a typical student taking this class.

Grade	A	B	C	D	F
Probability					

6. In each of the parts 1 – 5, explain if you used the classical, subjective, or frequency viewpoint in assigning the probabilities.

A MORE FORMAL LOOK AT PROBABILITY

In this topic, we have discussed probability in an informal way. We assign numbers called probabilities to outcomes in the sample space such that the sum of the numbers over all outcomes is equal to one. Here we look at probability from a more formal viewpoint. We define probability as a function on events that satisfies three basic laws or axioms. Then it turns out that all of the important facts about probabilities, including some facts that we used above, can be derived once we are given these three basic axioms.

Suppose that we write the sample space for our random experiment as S. An event, represented by a capital letter such as A, is a subset of S. Events, like sets, can be combined in various ways. We write

- $A \cap B$ as the event that both A and B occur (the *intersection* of the two events)
- $A \cup B$ as the event that either A or B occur (the *union* of the two events)
- $\bar{A}$ as the event that A does not occur (the *complement* of the event A)

To illustrate these set operations, suppose you choose a student at random from your class and record the month when she or he was born. The student could be born during 12 possible months and the sample space S is the list of these months:

S = {January, February, March, April, May, June, July, August, September, October, November, December}.

Suppose event L is the event that the student is born during the last half of the year and F is the event that the student is born during a month that is four letters long. So

L = { July, August, September, October, November, December }, and
F = {June, July}.

We can illustrate various set operations:

- $L \cap F$ is the event that the student is born during the last half of the year AND is born in a four-letter month = {July}.
- $L \cup F$, in contrast, is the event that the student is EITHER born during the last half of the year OR born in a four-letter month = {June, July, August, September, October, November, December}.
- $\bar{L}$ is the event that the student is NOT born during the last half of the year = { January, February, March, April, May, June}

PRACTICE: SET OPERATIONS

Suppose in a simple lottery game, you choose two different numbers from the set {1, 2, 3, 4, 5}. The order that you choose the numbers is not important.

1. Write down the sample space *S* (you should have 10 different outcomes).

2. Let *O* denote the event that you choose the number 1, and let *S* denote the event that the sum of the two numbers is equal to six. Write down the set of outcomes for *O* and the set of outcomes for *S*.

3. Write down the outcomes in

(a) $O \cap S$

(b) $O \cup S$

(c) $\bar{O}$

4. We say that two events are mutually exclusive if they have no outcomes in common. Are *O* and *S* mutually exclusive? Why or why not?

5. Find an event *A* such that *A* and *O* are mutually exclusive.

The Three Probability Axioms

Now that we have set up a sample space S and events, we can define probabilities that are numbers assigned to events. There are three basic laws or axioms that define probabilities:

- **Axiom 1**: For any event A, $P(A) \geq 0$. (That is, all probabilities are nonnegative values.)
- **Axiom 2**: $P(S) = 1$ (That is, the probability that you observe something in the sample space is one.)
- **Axiom 3**: Suppose you have a sequence of events $A_1, A_2, A_3, \ldots$ that are mutually exclusive. (This means that for any two events in the sequence, say A_2 and A_3, the intersection of the two events is the empty set – that is, $A_2 \cap A_3 = \varnothing$.) Then you find the probability of the union of the events by adding the individual event probabilities:

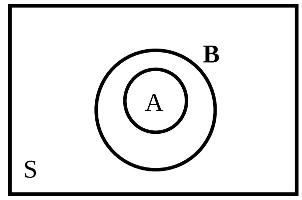
$$P(A_1 \cup A_2 \cup A_3 \cup \cdots) = P(A_1) + P(A_2) + P(A_3) + \cdots$$

Given the three basic axioms, we can prove some additional facts about probabilities. We call these additional facts *properties* – these are not axioms, but rather additional facts that can be derived knowing the axioms. Below we state several familiar properties about probabilities and prove how each property follows logically from the axioms.

Property 1: A subset of B $A \subset B$, $P(A) \leq P(B)$

This property says that if you have two sets, such that one set is a subset of another set, then the probability of the first set can't exceed the probability of the second set. This may seem pretty obvious, but how can we prove this from the axioms?

Proof: We start with a Venn diagram where a set A is a subset of set B.



Note that we can write the larger set B as the union of A and $\bar{A} \cap B$ -- that is,

$$B = A \cup (\bar{A} \cap B)$$

Note that $A \cap B$ and $\bar{A} \cap B$ are mutually exclusive (they have no overlap). So we can apply Axiom 3 and write

$$P(B) = P(A) + P(\bar{A} \cap B)$$

Also, by Axiom 1, the probability of any event is nonnegative. So we have shown that the probability of B is equal to the probability of A plus a nonnegative number. So this implies

$$P(B) \geq P(A),$$

which is what we wish to prove.

Property 2: $P(A) \leq 1$

This is pretty obvious – we know probabilities can't be larger than 1. But how can we prove this given our known facts that include the axioms and Property 1 that we just proved?

Proof: Actually this can be shown as a consequence of Property 1. Consider the two sets A and the sample space S. Obviously A is a subset of the sample space – that is,

$$A \subset S$$

So applying Property 1,

$$P(A) \le P(S) = 1$$

(We know P(S) from the second axiom.) So we have proved our result.

PRACTICE: PROVING SEVERAL PROPERTIES OF PROBABILITIES.

In this practice, we will outline the proofs of several properties of probabilities. Each step in the proof is written down and you are asked to justify why this step is true.

Property 3: $P(\varnothing) = 0$

This third property says that the probability of the empty set (the event consisting of no members) is equal to zero.

Proof: In the following, we outline the steps of the proof and you are asked to give the rationale for each step.

Step 1: $\varnothing = \varnothing \cup \varnothing$

Why is this true?

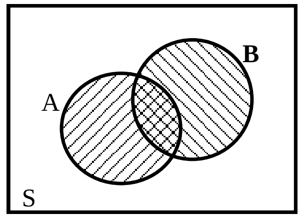
Step 2: $P(\varnothing) = P(\varnothing) + P(\varnothing)$

Why is this true?

Step 3: $P(\varnothing) = 0$

Why is this true?

Property 4: $P(A) = P(A \cap B) + P(A \cap \bar{B})$



A Venn diagram showing the two sets A and B is shown above.

Step 1: Write set A as the union of two sets that are mutually exclusive.

Step 2: Apply Axiom 3 to the union statement in Step 1.

THE COMPLEMENT AND ADDITION RULES

There are two additional properties of probabilities that we will find useful in computation. Both of these properties will be stated without proof, but an outline of the proofs will be given in the exercises.

The first property, called the *complement rule*, says that the property of the complement of an event is simply one minus the probability of the event.

Complement rule: For an event *A*, $P(\bar{A}) = 1 - P(A)$.

The second property, called the *addition rule*, gives a formula for the probability of the union of two events.

Addition rule: For two events *A* and *B*, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Both of these rules are best illustrated by an example. Let us revisit our example where we are interested in the birth month of a child selected from our class. As before, we let L represent the event that the student is born during the last half of the year and F denote the event that the student is born during a month that is four letters long.

There are 12 possible outcomes for the birth month. One could assume that each month is equally likely to occur, but actually in the U.S. population, the numbers of births during the different months do vary. Using data from the births in the U.S. in 1978, we obtain the following probabilities for the months. We see that August is the most likely birth month with a probability of .091 and February (the shortest month) has the smallest probability of .075.

MONTH	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
PROB	.081	.075	.083	.076	.082	.081	.088	.091	.088	.087	.082	.085

Using this probability table, we find

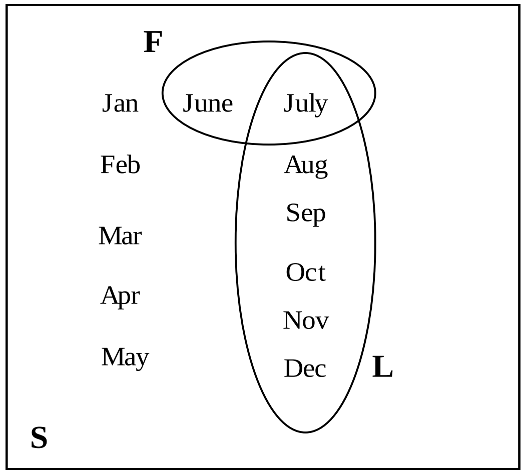
1. P(L) = P(July, August, September, October, November, December)
   = .088 +.091+.088+.098+.082+.085 = .521.
2. P(F) = P(June, July) = .081+.088 = .169.

Now we are ready to illustrate the two probability rules.

What is the probability the student is *not* born during the last half of the year? We could find this by summing the probabilities of the first six months of the year. It is easier to note that we wish to find the probability of the complement of the event L, and we apply the complement rule to find the probability.

$$P(\bar{L}) = 1 - P(L) = 1 - .521 = .479.$$

What is the probability the student is either born during the last six months of the year *or* a month four letters long? In the below figure, we show the sample space S consisting of the twelve possible birth months and the sets F and L are shown by circling the relevant outcomes. We wish to find the probability of the set $F \cup L$ which is the union of the two circled sets.



Applying the addition rule, we find the probability of $F \cup L$ by adding the probabilities of F and L and subtracting the probability of the intersection event $F \cap L = \{July\}$:

$$P(F \cup L) = P(F) + P(L) - P(F \cap L)$$
$$= .521 + .169 - .088 = .602$$

Looking at the figure, the formula should make sense. When we add the probabilities of the events F and L, we add the probability of the month July twice, and to get the correct answer, we need to subtract the outcome (July) that is common to both F and L.

SPECIAL NOTE: Is it possible to simply add the probabilities of two events, say A and B, to get the probability of the union $A \cup B$? Suppose the sets A and B are mutually exclusive which means they have no outcomes in common. In this special case, $A \cap B = \varnothing$, $P(A \cap B) = 0$, and $P(A \cup B) = P(A) + P(B)$. For example, suppose you are interested that your student is born in the last half the year (event L) or in May (event M). Here it is not possible to be born in the last half of the year and in May so $L \cap M = \varnothing$. In this case, $P(L \cup M) = P(L) + P(M) = .521 + .082 = .603$.

PRACTICE: THE ADDITION AND COMPLEMENT RULES

Consider again the simple lottery game where you choose two numbers (without replacement} from the set {1, 2, 3, 4, 5}. There are 10 possible outcomes shown below.

{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 3}, {2, 4}, {2, 5}, {3, 4}, {3, 5}, {4, 5}

Here each outcome is equally likely, so we can assign a probability of 1/10 to each. As before, let O denote the event that the number 1 is chosen, and S denote the event that the two numbers chosen sum to 6.

1. Find P(O) and P(S).
2. Using the complement rule, find the probability $P(\bar{O})$.
3. Are O and S mutually exclusive events? Why or why not?
4. Use the addition rule to find $P(O \cup S)$.
5. Find an event C such that events C and S are mutually exclusive.

WRAP-UP

In this topic, we started talking about assigning probabilities. First one writes down a list of all possible outcomes, the sample space, and then one assigns probabilities to the different outcomes. There are basic rules, called axioms, that all probabilities must follow. The probability of any event must be nonnegative, the probability of the sample space is equal to 1, and the probability of a union of mutually exclusive events is the sum of the probabilities of the events. There are additional facts about probabilities, called properties that can be derived from the axioms. Generally, probabilities are difficult to specify, but one can assign reasonable sets of probabilities given information for a particular problem.

EXERCISES

1. **Writing Sample Spaces**

For the following random experiments, give an appropriate sample space for the random experiment. You can use any method (a list, a tree diagram, a two-way table) to represent the possible outcomes.

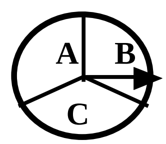
a. You simultaneously toss a coin and roll a die.

b. Construct a word from the five letters a, a, e, e, s

c. Suppose a person lives at point 0 and each second she randomly takes a step to the right or a step to the left. You observe the person's location after four steps.
d. In the first round of next year's baseball playoff, the two teams, say the Phillies and the Diamondbacks play in a best-of-five series where the first team to win three games wins the playoff.
e. A couple decides to have children until a boy is born.
f. A roulette game is played with a wheel with 38 slots numbered 0, 00, 1, …, 36. Suppose you place a $10 bet that an even number (not 0) will come up in the wheel. The wheel is spun.
g. Suppose three batters, Marlon, Jimmy, and Bobby, come to bat during one inning of a baseball game. Each batter can either get a hit, walk, or get out.

2. **Writing Sample Spaces**

For the following random experiments, give an appropriate sample space for the random experiment. You can use any method (a list, a tree diagram, a two-way table) to represent the possible outcomes.
a. You toss three coins.
b. You spin the spinner (shown to the right) three times.



c. When you are buying a car, you have a choice of three colors, two different engine sizes, and whether or not to have a CD player. You make each choice completely at random and go to the dealership to pick up your new car.
d. Five horses, Lucky, Best Girl, Stripes, Solid, and Jokester compete in a race. You record the horses that win, place, and show (finish first, second, and third) in the race.
e. You and a friend each think of a whole number between 0 and 9.
f. On your computer, you have a playlist of 4 songs denoted by a, b, c, d. You play them in a random order.
g. Suppose a basketball player takes a "one-and-one" foul shot. (This means that he attempts one shot and if the first shot is successful, he gets to attempt a second shot.)

3. **Writing Sample Spaces**

For the following random experiments, give an appropriate sample space for the random experiment. You can use any method (a list, a tree diagram, a two-way table) to represent the possible outcomes.

a. Your school plays four football games in a month.

b. You call a "random" household in your city and record the number of hours that the TV was on that day.

c. You talk to an Ohio resident who has recently received her college degree. How many years did she go to college?

d. The political party of our next elected U.S. President.

e. The age of our next President when he/she is inaugurated.

f. The year a human will next land on the moon.

4. **Writing Sample Spaces**

For the following random experiments, give an appropriate sample space for the random experiment. You can use any method (a list, a tree diagram, a two-way table) to represent the possible outcomes.

a. The time you arrive at your first class on Monday that begins at 8:30 AM.

b. You throw a ball in the air and record how high it is thrown (in feet).

c. Your cost of textbooks next semester.

d. The number of children you will have.

e. You take a five question true/false test.

f. You drive on the major street in your town and pass through four traffic lights.

5. **Probability Assignments**

Give reasonable assignments of probabilities based on the given information.

a. In the United States, there were 4058 thousand babies born in the year 2000 and 1980 thousand were girls. Assign probabilities to the possible genders of your next child.

Gender	Boy	Girl
Probability		

b. Next year, your school will be playing your neighboring school in football. Your neighboring school is a strong favorite to win the game.

Winner of game	Your school	Your neighboring school
Probability		

c. You have an unusual die that shows 1 on two sides, 2 on two sides, and 3 and 4 on the remaining two sides.

Roll of die	1	2	3	4	5	6
Probability						

6. **Probability Assignments**

Based on the given information, decide if the stated probabilities are reasonable. If they are not, explain how they should be changed.

a. Suppose you play two games of chess with a chess master. You can either win 0 games, 1 game, or 2 games, so the probability of each outcome is equal to 1/3.

b. Suppose 10% of cars in a car show are Corvettes and you know that red is the most popular Corvette color. So the chance that a randomly chosen car is a red Corvette must be larger than 10%.

c. In a Florida community, you are told that 30% of the residents play golf, 20% play tennis, and 40% of the residents play golf and tennis.

d. Suppose you are told that 10% of the students in a particular class get A, 20% get B, 20% get C, and 20% get D. That means that 30% of the class must fail the class.

7. **Finding the Right Key**

Suppose your key chain has five keys, one of which will open up your front door of your apartment. One night, you randomly try keys until the right one is found. Here are the possible numbers of keys you will try until you get the right one:

1 key, 2 keys, 3 keys, 4 keys, 5 keys

a. Circle the outcome that you think is most likely to occur.

1 key, 2 keys, 3 keys, 4 keys, 5 keys

b. Circle the outcome that you think is least likely to occur.

1 key, 2 keys, 3 keys, 4 keys, 5 keys

c. Based on your answers to parts a and b, assign probabilities to the six possible outcomes.

Outcome	1 key	2 keys	3 keys	4 keys	5 keys
Probability					

8. **Playing Roulette**

One night in Reno, you play roulette five times. Each game you bet $5 – if you win, you win $10; otherwise, you lose your $5. You start the evening with $25. Here are the possible amounts of money you will have after playing the five games.

$0, $10, $20, $30, $40, $50 .

a. Circle the outcome that you think is most likely to occur.

$0, $10, $20, $30, $40, $50

b. Circle the outcome that you think is least likely to occur.

$0, $10, $20, $30, $40, $50

c. Based on your answers to parts a and b, assign probabilities to the six possible outcomes.

Outcome	$0	$10	$20	$30	$40	$50
Probability						

9. **Cost of Your Next Car**

Consider the cost of the next new car you will purchase in the future. There are five possibilities:

cheapest: the car will cost less than $5000

cheaper: the car will cost between $5000 and $10,000.

moderate: the car will cost between $10,000 and $20,000

expensive: the car will cost between $20,000 and $30,000

really expensive: the car will cost over $30,0000

a. Circle the outcome that you think is most likely to occur.

cheapest, cheaper, moderate, expensive, really expensive

b. Circle the outcome that you think is least likely to occur.

cheapest, cheaper, moderate, expensive, really expensive

c. Based on your answers to parts a and b, assign probabilities to the five possible outcomes.

Outcome	cheapest	cheaper	moderate	Expensive	Really expensive
Probability					

10. **Flipping a Coin**

Suppose you flip a coin twice. There are four possible outcomes (H stands for heads and T stands for tails).

HH, HT, TH, TT

a. Circle the outcome that you think is most likely to occur.

HH, HT, TH, TT

b. Circle the outcome that you think is least likely to occur.

HH, HT, TH, TT

c. Based on your answers to parts a and b, assign probabilities to the four possible outcomes.

Outcome	HH	HT	TH	TT
Probability				

11. **Playing Songs in Your IPod**

Suppose you play three songs by Jewell (J), Madonna (M), and Plumb (P) in a random order.

a. Write down all possible ordering of the three songs.

b. Let M = event that the Madonna song is played first and B = event that the Madonna song is played before the Jewell song. Find P(M) and P(B).

c. Write down the outcomes in the event $M \cap B$ and find the probability $P(M \cap B)$

d. By use of the complement rule, find $P(\overline{B})$.

e. By use of the addition rule, find $P(M \cup B)$.

12. **Student of the Day**

Suppose that students at a local high school are distributed by grade level and gender by the following table.

	Freshmen	Sophomores	Juniors	Seniors	TOTAL
Male	25	30	24	19	98
Female	20	32	28	15	95
TOTAL	45	62	52	34	193

Suppose that a student is chosen at random from the school to be the "student of the day". Let F = event that student is a freshmen, J = event that student is a junior, and M = event that student is a male.

a. Find the probability $P(\overline{F})$.

b. Are events F and J mutually exclusive. Why?

c. Find $P(F \cup J)$.

d. Find $P(F \cap M)$

e. Find $P(F \cup M)$

13. **Proving Properties of Probabilities**

Given the three probability axioms and the properties already proved, prove the complement rule $P(\bar{A}) = 1 - P(A)$. An outline of the proof is written below.

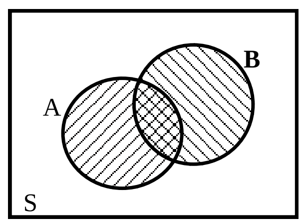
a. Write the sample space S as the union of the sets A and $\bar{A}$.

b. Apply Axiom 3.

c. Apply Axiom 2.

14. **Proving Properties of Probabilities**

Given the three probability axioms and the properties already proved, prove the addition rule $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. A Venn diagram and an outline of the proof are written below.



a. Write the set $A \cup B$ as the union of three sets that are mutually exclusive.

b. Apply Axiom 2 to write $P(A \cup B)$ as the sum of three terms.

c. Write the set A as the union of two mutually exclusive sets.

d. Apply Axiom 2 to write *P(A)* as the sum of two terms.

e. By writing B as the union of two mutually exclusive sets and applying Axiom 2, write *P(B)* as the sum of two terms.

f. By making appropriate substitutions to the expression in part b, one obtains the desired result.